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A CONCEPTUAL STUDY OF NORMAL SUBGROUPS & QUOTIENT GRUPS IN THEORY

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ABSTRACT

This paper depicts the path in which the concept of remainder gather was found and created amid the nineteenth century, and analyzes conceivable purposes behind this improvement. The commitments of seven mathematicians specifically are examined: Galois, Betti, Jordan, Dedekind, Dyck, Frobenius, and Ho1der. The essential connection between the advancement of this concept and the abstraction of gathering hypothesis is considered.

1. INTRODUCTION

In spite of the fact that the concept of remainder gather is currently thought to be major to the investigation of gatherings it is a concept which was obscure to early gathering scholars. It developed relatively late ever: around the finish of the nineteenth century. The fundamental purpose behind this deferral is that with a specific end goal to give a conspicuously current meaning of a remainder gathering, it is important to consider bunches in an abstract way. In this manner the advancement of the concept of remainder amasses is firmly connected with the abstraction of gathering hypothesis. This procedure of abstraction occurred for the most part amid the period 1870-1890 and was done solely by German mathematicians. Therefore by 1890 the advancement and comprehension of the concept of remainder gather had generally been finished [1].

The commitments of seven mathematicians to the development of this concept are considered here. An early comprehension of the concept inside Galois Theory and change aggregate hypothesis can be found in looks into of Galois, Betti, and Jordan (work which falls inside the period 1829-1873). A few investigations by Dedekind amid the 1850s uncover his astounding handle of abstract gathering hypothesis and his reasonable comprehension of the concept of remainder gathering. In crafted by Dyck, Frobenius, lastly Holder, the concept was investigated amid the 1880s inside abstract gathering hypothesis and after this time it quickly picked up acknowledgment in the scientific group. What is a remainder gathering? Is there just a single 'right' definition? In the event that there is, it ought to be conceivable to follow the advancement of the concept considering this definition and to quantify early endeavors at a definition against it. There is absolutely a standard present day definition, utilized by such reading material as [2]:

For a gathering G, the remainder assemble G/H is the arrangement of cosetsHx $(x \in G)$ of the normal subgroup H of G, with multiplication given by Hx_1Hx_2 " $= Hx_1\ x_2$ $(x_l, x_2 \in G)$.

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This definition could have been given regarding left cosetsxH, as it is in Rose, however such a refinement is repetitive since a typical subgroup H of G is a subgroup for which Hx = xH for all $x \in G$. Why has this turned into the standard definition? In what ways does it typify the concept of remainder aggregate well? I recommend that it does as such in two ways. In the first place, this definition makes utilize just of the components of the gathering G itself, with these components being consolidated especially. We don't need to utilize any concepts 'outside' the gathering. Second, the definition does not rely upon speaking to G in any restricted: it is ~abstract' and can be connected to any gathering. The burden of this definition is that it is difficult to sum up to other arithmetical structures [3].

Notwithstanding, a minute's idea uncovers that the essential thought behind this definition is that of identicalness. For any logarithmic structure, in the event that we can isolate its components into proportionality classes and create a very much characterized 'duplication' of these classes, we have framed a remainder structure. What must one comprehend keeping in mind the end goal to give a meaning of remainder assemble which fulfills the two criteria said above? For the principal, we should perceive that the components of a remainder aggregate are not of an indistinguishable sort from the components of the first gathering. In the above definition they are sets of the first components: these sets are the ordinary subgroup H and its cosets. For the second, we should (clearly) comprehend the abstract idea of a gathering [4].

Before the finish of the most recent century, the 'standard' definition expressed above had been figured and was being used close by before, more change theoretic, endeavors at a definition. I will likely overview the advancement of the concept up to that time in the light of the two criteria for definition said above. I am not, in any case, rejecting the bits of knowledge of those mathematicians who communicated the concept in different ways and who in this way don't meet these criteria, either through their own decision or through the limitations of the techniques accessible to them. To start with let us take a gander at the ancient times of this improvement [5].

2. IMPLICIT USE OF THE CONCEPT

GALOIS With the advantage of insight into the past we can see that the concept of remainder aggregate was available on many events previously an express definition was given. One thought which would now be able to be comprehended as far as remainder bunches is found in the Galois Theory of logarithmic conditions. In his unique clarification of the hypothesis, Galois made express out of the blue the concepts of gathering (the gathering) and typicality of a subgroup (clean disintegration). He examined how a given condition can be fathomed by exploring the structure of its related gathering. He utilized 'le gathering' to allude to an arrangement of game plans of the underlying foundations of the condition as opposed to an arrangement of changes of these courses of action, to which it later came to allude. He did, be that as it may, comprehend that it is the stages which have the 'gathering structure [6].'

In present day terms, this comment expresses that if the gathering G of a condition has a typical subgroup H, the

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condition can be fathomed by methods for two conditions whose gatherings we know as G/H and H. Since Galois had no concept of remainder gathering, the gathering that would now be called G/H was to him the gathering related with his 'assistant condition.' This momentous understanding lies at the core of Galois Theory; the entry cited above is an especially clear detailing of Galois' Proposition III in the Premier Memoire. Galois' contemplations were fixated on finding a strategy for settling on the resolvability of a condition by radicals, so he didn't examine how the gathering of such an assistant condition emerges from the gathering of the first condition [7].

3. EARLY UNDERSTANDING OF THE CONCEPT: BETTI

The principal broad analysis on Galois' works to be distributed was endeavored by Betti. The two areas of this paper are committed to the hypothesis of substitutions and of substitution gatherings and to Galois' hypothesis of conditions. Betti knew during this season of the advances in substitution aggregate hypothesis as set out in Serret's reading material yet did not say Cauchy's work of 1845 and 1846 regarding this matter.

In this manner Betti was looking to clarify Galois' work from the perspective of substitution bunch hypothesis. Integral to his approach was an examination of the path in which the gathering of an assistant condition is identified with the gathering of the first condition. The primary portion of his paper was pointed (to a limited extent) at the treatment of this inquiry from an absolutely aggregate theoretic perspective. That is, Betti was looking for an approach to clarify the way that a typical subgroup of a gathering offers ascend to another gathering - which we now comprehend as a remainder amass - and he was trying to do this inside substitution assemble hypothesis [8].

Ahead of schedule in the paper Betti presented the possibility of conjugation of substitutions which in his terminology is called deduction. After a concise clarification of what is implied by a gathering of plans and its substitutions he investigated conjugation of gatherings. He considered two gatherings ¢) and 1 b, both following up on a similar accumulation of' amounts'; the gathering G is conjugated by the substitutions of I ~. So he limited the hypothesis to the circumstance when the 'conjugating' substitutions frame a gathering.

Betti considered two cases: the principal happens when none of the (nonidentity) substitutions of F standardizes 0 and the second when F is contained in the normalizer of ¢). No middle of the road case was talked about. In the two cases Betti developed the 'item' $\hat{G}\Gamma$ ~ "= $\{\theta_j\ \Psi_i\theta_j \in \hat{G}, \Psi_i \in \Gamma\}$and considered what might happen if the gathering of substitutions in $\hat{G}\Gamma$ framed a gathering $\hat{H}$. In the second case a gathering is certainly shaped and G turns into an ordinary subgroup of $\hat{H}$. Indeed $\hat{H}$is the part augmentation of $\hat{G}$by Γ ~ if $\hat{G}\cap\Gamma = \{1\}$. Betti determined here that $\hat{G}\cap\Gamma$ ~ = {1} and later managed the circumstance when the substitutions of these gatherings are not particular. He alluded to F as a multiplier of $\hat{G}$ and as a divisor of H. His decision of these terms must reflect to some degree the path in which he saw the connection amongst $\hat{H}$and Γ/. We realize that in this split augmentation Γ~ is isomorphic to the remainder gathering $\hat{G}/\hat{H}$.In any case, Γ ~ is additionally a subgroup of, Γ. How did Betti consider the relationship?

Betti built up the hypothesis further to frame a gathering which we know as the picture of the stage portrayal of$\hat{G}$on the privilege cosets of $\hat{G}$. The name Betti provided for it (he characterized it as an accumulation K of courses of action) was the gathering oj game plans on the conjugates. At the point when the substitutions of F standardize G (and Betti determined that the substitutions of these two gatherings are unmistakable), the picture k of this stage portrayal is (isomorphic to) the remainder gathering $\hat{G}/\hat{H}$and along these lines is isomorphic to F. Betti's disarray in building up his hypothesis just for the situation when each remainder gathering of a given gathering is isomorphic to a subgroup of that gathering could subsequently have been kept away from, had he made utilization of $\hat{G}/\hat{H}$ as opposed to I ~ when examining the connection between a gathering/and an ordinary subgroup O [9].

A couple of pages later on in his paper Betti encountered the concept of remainder bunch once more. He considered what might happen if one somehow happened to frame the result of two gatherings whose crossing point isn't unimportant. He changed his notation with the goal that the group Γ moved toward becoming Γ_n'. Betti attested (in present day terms) that in the event that one substitution of Γ_n is the result of another by a substitution of $\hat{G}$ ($\gamma_1 = g\gamma_2$ for $g \in G$ and $\gamma_1, \gamma_2 \in \Gamma$), at that point γ_1 and γ_2 are in the same coset of $\hat{G}$. Betti composed that these two substitutions of Γ_n must be considered as equivalent to each other in Γ_n. Essentially he was characterizing an equivalence connection on the substitutions of Γ_n. In this way the request of Γ_n is given by the quantity of its substitutions which are in various cosets of $\hat{G}$. In current notation the request of Γ_n is given by $[\Gamma_n, : (\hat{G} \cap \Gamma_n,]$. These days we would think about the equivalence classes characterized here as components of the remainder bunch $\Gamma_n/(\hat{G} \cap \Gamma_n)$. Truth be told this was the approach that Jordan took to present the concept of remainder assemble 20 years after Betti's work. Betti, in any case, did not assume that he was managing another gathering yet rather incorporating an additional condition in the meaning of Γ_n [10].

Betti at that point clarified that (in current terms) on the off chance that we pick an arrangement of coset representatives for the cosets of $\hat{G}$ (these cosets are given by duplicating by the substitutions of Γ_n,), we will be unable to pick the representatives to frame a gathering. It appears that Betti was recognizing here that not all expansions split. He understood, be that as it may, this did not influence the hypothesis he had created. So Betti appears to have seen a few yet not the greater part of the thoughts behind that of remainder gathering. He understood that in uncommon cases one can 'separate' one gathering by another and that the aftereffect of this division will be a third gathering. Nonetheless, he didn't understand that, not at all like division in the genuine numbers where the outcome likewise lies in the genuine numbers, the aftereffect of this division is a gathering which acts in an unexpected way. It can't be contrasted and the first two gatherings (as gatherings of substitutions following up on an arrangement of game plans) in a straightforward way.

4. A SYSTEMATIC APPROACH TO THE CONCEPT: JORDAN

So we go to the following mathematician in the story, Camille Jordan, to whom a few observers have ascribed the principal unequivocal meaning of a remainder gathering. Specifically Gaston Julia, who composed the prelude to Jordan's Oeuvres, says: Although, similar to his counterparts, Jordan considers just gatherings of permutations.... it is he who, be that as it may, depicts the "abstract" idea of remainder gathering. The reality of the matter is that Jordan just considered gatherings as gatherings of substitutions or changes, even in fill in as late as 1917, despite the fact that he was no uncertainty acquainted with advancements in abstract gathering hypothesis. It is additionally evident that he characterized a gathering which is certainly isomorphic to that given by our 'standard' definition. Furthermore, in one sense Jordan brought 'out the "abstract" thought of remainder gathering,' in that he saw how the 'abstract' concept functions inside change assemble hypothesis [11].

Dieudonne delivered a few notes on Jordan's work in limited gathering hypothesis which were distributed toward the start of the Oeuvres. He examined the hypothesis of Jordan's which Holder later reached out to wind up what is presently known as the Jordan-Holder Theorem. The hypothesis expresses that for a limited gathering G, any two composition arrangement for G have a similar length and their composition factors, aside from the request in which they happen, are isomorphic. Jordan demonstrated that the requests of these composition factors (thought of as the proportions of the requests of progressive gatherings in a composition arrangement) are the same for any two composition arrangement. The confirmation of this hypothesis initially showed up in Jordan's Trait [12].

It will be seen that this thought of figuring 'module' a typical subgroup is in fact the thought which offered ascend to Jordan's concept of remainder gathering. In his approach there is an undeniable parallel with Gauss' work on number juggling congruencies in 1801 which Jordan no uncertainty had as a primary concern. Jordan utilized the image ~ which Gauss had acquainted with indicate compatibility. Dieudonne remarked on the Jordan-Ho1der Theorem that it would accept its conclusive frame just with Holder. This could likewise be said of the concept of remainder gathering: in the two cases Jordan's thoughts were early definitions which later offered route to the acknowledged 'standard' structures.

Jordan attempted to build up Galois' thoughts in the 1860s and the concept of remainder bunch in this way showed up verifiably in his examination as the gathering of an assistant condition. He comprehended and clarified the strategies for Galois Theory more unmistakably than prior mathematicians and built up the hypothesis of substitution gatherings. In any case, he made no endeavor to deliver a remainder bunch unequivocally.

Along these lines Jordan's remainder aggregate structure ~ comprises of coinciding classes of the components s_1 s_2 ....from which G is shaped. His notation was composed so that, in actuality, the remainder amass comprises of one representative from every contumacy class, despite the fact that these representatives don't really shape a gathering themselves [13].

Jordan made utilization of these new remainder bunches in another paper of 1873, "Moiré on primitive gatherings". In this long paper, he connected the level of certain

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primitive substitution bunches with properties of a specific substitution in each gathering. It appears to be far-fetched that the references to his remainder gatherings, which are installed somewhere down in this paper, would have had much impact on Jordan's counterparts. This paper seems, by all accounts, to be toward the end in which Jordan utilized the new concept. In any case, the first of these two papers marks noteworthy advance in its improvement. The paper was referred to a few times by Frobenius in his later work on remainder gatherings and furthermore by Burkhardt in a reference book article entitled "Finite discrete groups"

5. ABSTRACTION AND THE CONCEPT OF EQUIVALENCE

After these papers of Jordan's the advancement turns out to be less simple to follow. The concept of remainder bunch started to be drawn closer from new and distinctive points. This procedure occurred as more abstract thoughts were brought into the investigation of gatherings. The old hypotheses were continuously being reformulated and stretched out by methods for these new thoughts. As these incredible changes were occurring, the convenience of the concept of equivalence was increasingly unmistakably perceived.

Frege's book "The basics of arithmetic"incorporates a nitty gritty discourse of the uniformity of numbers, which he characterized by methods for one-one correspondence. The definition relies upon the way that one-one correspondence is an equivalence connection and, despite the fact that Frege did not give an express proclamation of the concept, he certainly perceived its significance. Dedekind happened upon the concept of equivalence because of his examinations concerning the establishments of investigation and in this manner into the genuine number framework [14].

Cantor likewise found the concept of equivalence in light of his work on transfinite numbers started in 1870 lastly summed up in two journals. He made utilization of one-one correspondence to characterize 'equivalence' of sets and by methods for this concept to characterize cardinal numbers. It might be that few of these mathematicians went over the possibility of equivalence freely.

6. PIONEER OF THE CONCEPT: DEDEKIND

Dedekind seems to have comprehended the part of equivalence at a substantially prior period, specifically in his work amid the years 1855-1858. He investigated the hypothesis of gatherings yet his compositions stayed unpublished until after his demise in 1916. Amid his lifetime he had done close to say this work to Frobenius in a letter of 1895. Dedekind investigated the concept of homomorphism in an area entitled ".equal von Gruppen." He framed a homomorphic picture M_1 of a gathering M by letting every component M_1 of M 'relate' to a component $N_1$of M_1, with specific conditions which we now perceive as the conditions for homomorphism. He demonstrated that M_1 is a gathering and that those components of M which 'compare' to the personality in M_1frame a subgroup N of M. He went ahead to find the concept of remainder gathering:

He communicated M as far as N and its cosets and expressed that a 'composition' of cosets can be characterized and that along these lines the cosets (he alluded to them basically as edifices, that is, 'sets') shape a gathering. There is a correspondence between the cosets and the components of M_1 with the end goal that to each coset compares one component of M_1, and to every component of Mt relates one coset [15].

7. THE INFLUENCES OF ABSTRACTION: DYCK AND FROBENIUS

We return now to the later period, to the year 1882, when a paper by Dyck, "Gruppen hypothetical Studied," showed up in Mathematical records. This paper has been examined by Chandler and Magnus. It starts with the abstract development of the free gathering, G say, on components A_1, A_2..... A_mDyck was worried about investigating the path in which any gathering $\hat{G}$, created by components A_1..... A_m (which are characterized by 'some foreordained procedure'), is identified with the first free gathering $\hat{G}$. Dyck's notation here as of now proposes a concealed supposition that $\hat{G}$ is a homomorphic picture of $\hat{G}$ under the homomorphism taking $\bar{A}_1$... ... .$\bar{A}_m$. To be sure, a couple of sentences later Dyck continued to demonstrate this is to be sure so and that there are two cases to consider. The main case is the point at which the gatherings are isomorphic; the second happens when any one component of $\hat{G}$ compares to boundlessly numerous components of $\hat{G}$. In the second case, he found the components of G which compare to the personality in $\hat{G}$ and demonstrated that they frame a typical subgroup $\hat{H}$ of $\hat{G}$,

$$\gamma' \cong \frac{\gamma}{g}.$$

The expression 'durch Adjunction von G' appears to have been acquired from Galois hypothesis, since one diminishes the Galois gathering of a condition to a typical subgroup by abutting components to the field, and these components are the foundations of a condition whose Galois assemble is $\hat{G}$. So the gathering $\hat{G}$can be thought of as having two factors: its typical subgroup $\hat{H}$and the homomorphic picture $\hat{G}$ which now, obviously, we likewise know as the remainder bunch $\hat{G}/\hat{H}$. Later in 1882 Dyck composed a further paper on amass hypothesis, "Gruppen theoretic Study II", situated to some extent on a few addresses he had as of late given. As he focused on that it is the 'abstract' properties of any gathering that are imperative.

The confirmation of the hypothesis for this case follows in a couple of lines from Frobenius' inductive theory. In current terms, Frobenius was shaping the remainder of $\wp$ by the cyclic typical subgroup produced by P. Sylow's own particular evidence of this hypothesis utilizes a thought from Galois hypothesis to build up what might now be found utilizing the concept of remainder gathering. He took a capacity y_0 of the letters on which the gatheringG acts and required that $y_0$stay invariant just under the substitutions of a subgroup G of request n^{α}", where n is prime and G has no subgroup of request n^{β} *for* $\beta > \alpha$. He considered how the unmistakable estimations of this capacity (relating to cosets of the subgroup) are permuted by the substitutions in the normalizer γ of g. We along these lines get a substitution amass 3" that is transitive and homomorphic to γ'. We would now compose

This strategy for Sylow's is proportional to shaping the stage portrayal on the cosets of g in its normalizer 3, The articles by Waterhouse, Scharlau, and Casadio and Zappa give definite investigations of the revelation of Sylow's hypotheses and the advancements in their verifications. One further paper ought to be said here: that composed by Capelli not long after Sylow's hypotheses were distributed. Capelli particularly planned to demonstrate the significance of isomorphism (a term which at that point secured both isomorphism and homomorphism) in the hypothesis of substitution gatherings. Throughout the paper he demonstrated the greater part of Sylow's hypotheses (however had clearly not perused Sylow's article), explored properties of gatherings of prime power arrange, and gave another confirmation of the Jordan-Holder Theorem. His investigation of the concept of isomorphism brought him to characterize a change portrayal similarly as Sylow did utilizing a capacity settled by the substitutions of a typical subgroup.

8. THE 'STANDARD' DEFINITION: HOLDER

The last stage, at that point, in this advancement accompanies Holder's paper Reduction of a self-assertive arithmetical condition to a chain of conditions" The inquiries which Holder wished to reply here are those incited by investigating Galois Theory in the light of abstract gathering hypothesis. Which bunches relate to the 'helper conditions' to what degree are these gatherings characterized what number of are there? The characteristic approach to answer these inquiries is to utilize the concept of remainder gathering. In the presentation Holder examined the straightforward gatherings emerging from a composition arrangement, which he named 'Factor gruppen,' and noticed that this concept of 'Factor gruppen' is "a gathering theoretic thought that has up to this point not been satisfactorily valued". He expressed that he would set out just the most basic gathering theoretic thoughts in the exchange that took after. It appears that Holder did not consider the concept of remainder gathering to be either another or a troublesome one. The initial segment of the paper is a gathering theoretic segment. Holder gave sayings for a limited gathering and specified typical subgroups and composition arrangement. He at that point talked about' Factor decomposition' is the file of a gathering in a composition arrangement in the former gathering of the arrangement, as characterized by Jordan. In present day terms these are the requests of the composition factors [16].

9. WIDER RECOGNITION OF THE CONCEPT: THE 1890s

It was in the 1890s; after this paper of Holder's that the thought of remainder aggregate started to be joined into monographs and reading material. Netto's book on the hypothesis of substitutions and its applications, first distributed in 1882, was later overhauled by Netto and converted into English by F. N. Cole. This English release "varies from the German version in numerous vital particulars," as Netto commented in an expansion to the Preface. He included that he had considered "the entire material which has amassed over the span of time since the principal appearance of the book." specifically the English version incorporates the concept of remainder bunch which is characterized as a stage portrayal on the cosets of an ordinary subgroup, as one would expect in a book dedicated to substitutions.

The second volume of Weber's Lehrbuch der Algebra starts with an abstract meaning of a gathering and, after areas on subgroups and ordinary subgroups, there is an exchange of the properties of subgroups and cosets and the concept of remainder amass is exhibited. It is characterized as the gathering of cosets of a typical subgroup and Weber painstakingly demonstrated that these cosets do in reality shape a gathering. He went ahead to demonstrate the Jordan-Holder Theorem similarly that Holder had demonstrated it in 1889, utilizing remainder gatherings.

10. CONCLUSION

In this way unmistakably we can't trait the improvement of the concept of remainder gathering to any one individual or any one time. Like every scientific thought the period from its first event in primitive shape to its full understanding and acknowledgment as ordinary is a long one and incorporates the commitments of numerous mathematicians. The sentiments of present day pundits on the issue appear to support their social and numerical foundations: in Bourbaki it is expressed that Jordan presented the idea, while van der Waerden says that the advanced comprehension of remainder aggregate is because of Holder and that Jordan had it certainly. Wussing takes the view that the possibility of remainder bunch was a finding from the abstract gathering concept. He at that point takes note of that Holder put an abstract meaning of a gathering toward the start of his 1889 paper, which proposes that he considers Holder to have presented the concept of remainder gathering.

REFERENCES

1. Young, J. W. A. 1893. On the determination of groups whose order is the power of a prime? American Journal of Mathematics 15, 124-178.
2. Macdonald, I. D. 1968. The theory of groups. Oxford: Oxford Univ. Press.
3. Kiernan, B. M. 1971. The development of Galois Theory from Lagrange to Artin. Archive for History of Exact Sciences 8, 40-154.
4. Jordan, C. 1961. Oeuvres de Camille Jordan, J. Dieudonn6, Ed., Vol. I. Paris: Gauthier-Villars
5. Heijenoort, J. van 1967. From Frege to Godel: A source book in mathematical logic, 1879-1931. Cambridge, MA: Harvard Univ. Press.
6. Gauss, C. F. 1801. DisquisitionesArithmeticae. Leipzig: G. Fleischer. English Trans, by A. A. Clarke, S. J., New Haven/London: Yale Univ. Press, 1966.
7. Chandler, B., & Magnus, W. 1982. The history of combinatorial group theory: A case study in the history of ideas. New York: Springer-Verlag.
8. Burnside, W. 1897. Theory of groups' offinite order. Cambridge: Cambridge Univ. Press.
9. Cayley, A. 1893. Note on the so-called quotient G/H in the theory of groups. American Journal of Mathematics 15, 387-388; The collected mathematical papers, A. R. Forsyth, Ed., Vol. XIII, No. 934, pp. 336-337. Cambridge: Cambridge Univ. Press, 1897.
10. Herstein, I. N. 1975. Topics in algebra, 2nd Ed. New York: Wiley.

11. Kiernan, B. M. 1971. The development of Galois Theory from Lagrange to Artin. Archive for History of Exact Sciences 8, 40-154.
12. Holder, I. D. 1889. The theory of Quotients group. Oxford: Oxford Univ. Press.
13. Rose, J. S. 1978. A course on group theory. Cambridge: Cambridge Univ. Press
14. Waterhouse, W. C. 1980. The early proofs of Sylow's theorem. Archive for History of Exact Sciences 21, 279-290.
15. Young, J. W. A. 1893. On the determination of groups whose order is the power of a prime? American Journal of Mathematics 15, 124-178.
16. Wussing, H. 1969/1984. Berlin: VEB English version Trans, by A. Shenitzer: The genesis of the abstract group concept. Cambridge, MA/London: MIT Press, 1984. Page references are to the German and English editions, respectively.