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# REVIEW ON DUALITY IN DIFFERENTIABLE AND NON-DIFFERENTIABLE MATHEMATICAL PROGRAMMING

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**ABSTRACT**

The principle of duality associates two projects, one of which is known as the primal problem and the other is called dual Problem, such that the presence of an ideal answer for one o f them ensures an ideal answer for the other. On the off chance that the primal problem is constrained minimization (or maximization), the dual is a constrained maximization (or minimization) problem. The duality comes about have turned out to be exceptionally helpful in the improvement o f numerical algorithms for tackling certain classes of optimization problem. The presence o f duality theory in nonlinear programming problem creates numerical algorithm as it gives appropriate halting rules to primal and dual problems. A nonlinear programming problem and its dual are said to be symmetric if the dual o f the dual is the original problem. The primary commitment o f this postulation is to contemplate optimality and duality, including blended sort symmetric and self duality in an assortment of mathematical programming, limiting to non-differentiable nonlinear programming, variational problems non-differentiable fractional programming, non-differentiable fractional minmax programming, continuous-time minmax programming and minimaxvariational problems.

## 1. INTRODUCTION

Mathematical programming possessed a status of logical field in its own particular right amid late 1940's and from that point forward it has experienced gigantic advancement. It is presently considered as a standout amongst the most energetic and energizing branches of modem mathematics having broad applications in different settings, for example, designing, financial matters and common sciences. An exceptionally regular case of a mathematical programming problem shows up in discovering minimum weight design of structure subject to constraints on stress and deflection.

The form of a mathematical programming problem is as follows,

(MP): Optimize (minimize/maximize)f(x).

Subject to

$g_i(x) \leq 0, i=1,2,3,\ldots,m,$

$h_j(x)=0, j=1,2,3,\ldots,k,$

$x \in X$

Here the function f and each fj and hj are genuine esteemed capacities characterized on n dimensional Euclidean space $R^n$ and $X \subseteq R^n$. This is alluded to as the general mathematical programming problem. The constraints, $g_i(x) \leq 0$, i = 1, 2…, m are alluded to as to as inequality constraints, the constraints $h_j(x) = 0$, j = 1, 2... k are called fairness constraints. The incorporation $x \in X$ is known as a conceptual constraints. On the off chance that the goal and imperative capacities are differentiable then we portray the above problem as differentiable program. On the off chance that the goal and the inequality constraints are relative capacity and X is a convex set, at that point the above problem is known as a convex programming problem.

## 2. DUALITY IN DIFFERENTIABLE MATHEMATICAL PROGRAMMING

Let $f: R^n \rightarrow R$ and $h_j : R^n \rightarrow R$ , (j = 1,2,…m)then consider the nonlinear programming issue:

(P): Min f(x)

subject to,

$h_j(x) \leq 0$ ,(j = l,2 ,...,m ).

For $\lambda \in R^m$ the Lagrangian dual for issue (P) is defined as

$$\text{(LD):} \quad \underset{\lambda \geq 0}{\text{Max}}\left(\underset{x \in R^n}{\text{Min}}\left(f(x) + \lambda^T h(x)\right)\right)$$

That is,

(LD): M inf(u) + $\lambda^T$ h(u)

Subject to,

$$f(u) + \lambda^T h(u) = \underset{x \in R^n}{\text{Min}}\ f(x) + \lambda^T h(x), \quad \lambda \in R^m_+$$

In the event that all the function f and hj : (j = 1,2,. ..,m) are the differentiable convex functions, at that point the issue (LD) is comparable to the following issue:

(WD): M ax f(x) + $\lambda^T$h (x)

Subject to

V (f(x) + $\lambda^T$h(x)) = 0, $\lambda \geq 0$, $\lambda \in R^m$.

This is nothing yet the Wolfe sort dual for the issue (P). Mangasaria explained by implies o f a case that certain duality theorems may not be legitimate if the goal or the constraint function is a summed up convex function. This spurred Mond and Weir to introduce an alternate dual for (P) as

(MWD): Maxf(x)

Subject to

$\nabla(f(x) + \lambda^T h(x)) = 0,$

$\lambda^T h(x) \geq 0,$

$\lambda \geq 0, \lambda \in R^m,$

**3. DUALITY IN NON-DIFFERENTIABLE MATHEMATICAL PROGRAMMING**

Mond considered the following class of non-differentiable mathematical programming problems:

(NP): Min $f(x) + (x^T Bx)^{1/2}$

subject to

$hj(x) \leq 0 (j = 1,2..,m)$

Here f and hj (j = 1,2,…,m)are twice differentiable functions from $R^n$ to R and B is an n X n positive semi definite (symmetric) matrix. It is expected that the functions f and hj (j = l,2,...,m) are convex functions. They built up a duality theorem between (NP) what's more, the following issue

(ND): Max $f(u) + y^T h(u) - u^T \nabla [f(u) + y^T h(u)]$

subject to,

$\nabla f(u) + \nabla y^T h(u) + Bw = 0,$

$w^T Bw \leq 1,$

$y \geq 0$ .

Advance on the lines o fMond and Weir, Chandra, Craven and Mond introduced another dual program:

(NWD): Max $f(u) - u^T \nabla [f(u) + y^T h(u)]$

subject to

$\nabla f(u) + \nabla y^T h(u) + Bw = 0,$

$y^T h(u) \geq 0,$

$w^T B w \leq 1,$

$y \geq 0.$

what's more, settled duality theorems by assuming the capacity $f(.) + (.)^T Bw$ to be pseudoconvex and $y^T h$ to be quasiconvex for every single attainable solution o f (NP) and (NWD).

Later Mond and Schechter supplanted the square root term by the standard term what's more, viewed as the nondiffemtiable nonlinear programming problems as:

$(NP)_1$ Min $f(x) + \|S_x\|p$

Subject to,

$hj(x) \leq 0, (j = l,2,...,m).$

Here f and hj (j = l,2,...,m) are twice differentiable convex functions from $R^n$ to R. The dual for $(NP)_1$ is the issue:

(NP), Max $f(u) + y^T h(u) + u^T S^T v$

Subject to,

$\nabla f(u) + \nabla y^T h(u) + v = 0,$

$\|v\|_q \leq 1,$

$y \geq 0,$

Where p and q are conjugate examples.

## 4. SYMMETRIC DUALITY IN DIFFERENTIABLE MATHEMATICAL PROGRAMMING

Consider a capacity f(x, y) which is differentiable in $x \in R^n$ and $y \in R^m$. Dantzig et al introduced the following pair of problems:

(SP): M in $( f (x ,y ) - y^T \nabla y f(x,y))$

Subject to

$\nabla_y f(x ,y ) \leq 0 ,$

$(x ,y ) \geq 0 .$

(SD): Max f (x ,y ) $-x^T$ ,f (x ,y )

Subject to

Vxf(x, y )> 0 ,

(X, y) > 0,

what's more, demonstrated the presence o f a typical optimal solution to the primal (SP) and (SD), when (i) an optimal solution ($x_0$ $y_0$) to the primal (SP) exists (ii) f is convex in x for every y, concave in y for every x and (iii) f, twice differentiable, has the property that at ($x_0$, yo) its matrix o f second partials as for y is negative definite.

Mond additionally gave the following detailing of symmetric dual programming problems:

(MSP): Min f(x, y) - $y^T\nabla_y$ f(x, y)

subject to

∇, f(x, y) ≤ 0.

x≥ 0 .

(MSD): Max f(x, y) - $x^T\nabla_x$ f(x, y)

subject to

$\nabla_x$f (x ,y ) ≥ 0 .

y ≥0.

It might be commented here that in, the constraints o f both (SP) and (SD) include x ≥ 0, y ≥ 0, yet in just x ≥ 0 is required in the primal and just y ≥ 0 in the dual.

## 5. SYMMETRIC DUALITY IN NON-DIFFERENTIABLE MATHEMATICAL PROGRAMMING

Let f(x,y) be genuine esteemed continuously differentiable function in x ϵ $R^n$ , y ϵ $R^n$. Chandra and Husain introduced the following pair of symmetric dual non-differentiable programs and demonstrated duality comes about assuming convexity-concavity conditions on the bit ftinction f(x,y):

(NP): Min f(x,y) - $y^T\nabla_y$f(x,y) + $(x^TBx)^{1/2}$

subject to

$-\nabla_y$f(x,y) + Cw ≥ 0,

$w^T Cw \leq 1$,

$(x,y) > 0$.

(ND): Max $f(x,y) - x^T \nabla_x$, $f(x,y) - (y^T Cy)^{1/2}$

subject to

$-\nabla$, $f(x,y) - Bz \leq 0$,

$z^T B z \leq 1$,

**$(x,y) \geq 0$.**

where B and C are n x m and m x m positive semi definite matrices.

Facilitate on the lines ofMond and Weir, Chandra, Craven and Mond exhibited another combine o f symmetric dual non-differentiable programs by wfeakening the convexity conditions on the bit function f(x,y) to pseudoconvexity

The problems considered in are:

(PS): Min $f(x,y) + (x^T Bx)^{1/2} - y^T Cz$

subject to

$\nabla_y f(x ,y ) - Cz < 0$,

$y^T[\nabla_y f(x,y) - Cz] > 0$,

$z^T Cz \leq 1$,

$x \geq 0$ .

(DS): Max $f(x,y) - (y^T Cy)^{1/2} + x^T Bw$

subject to

$\nabla_x f(x,y) + Bw \geq 0$,

$x^T[\nabla_x f(x,y) + B w ] \leq 0$,

$w^T B w \leq 1$,

$y \geq 0$.

## 6. DIFFERENTIABLE FRACTIONAL PROGRAMMING

Let f, - g and $h_j$ (J = 1, 2,m) be genuine esteemed differentiable convex functions characterized on an open convex set $X \subseteq R^n$ . At that point consider the convex-concave fractional programming problem;

(FP):

Min $\frac{f(x)}{g(x)}$

Subject to

$h_j \leq 0$, (j = 1,2,…,m)

Where S = {x ϵ X :$h_j$ (x) ≤ 0, j = 1,2,....,m}, g(x) >0 for all x ϵ X, and if g(x) is not

affine then f(x) ≥ 0 for all x ϵ X.

Two models o f duality for (FP) are outstanding and these have been generally talked about in the writing. These are because of Bector ,Jagannathgan and Schaible . The Bector's double [17] for (FP) is the problem

$$\text{(BD):} \quad \text{Max } \frac{f(u) + y^T h(u)}{g(u)}$$

Subject to,

$$\nabla \left( \frac{f(u) + y^T h(u)}{g(u)} \right) = 0,$$

$$y \geq 0$$

This is basically the Wolfe double for the accompanying proportionate problem (EPF) of FP:

(EFP) : Min $\frac{f(x)}{g(x)}$

Subject to,

$$\frac{hj(x)}{g(x)}$$

In (BD) to make the target function pseudoconvex, with the goal that duality theorems hold, one requires that f(u)+ $y^T$h(u) ≥ 0, if g isn't relative.

## 7. NON-DIFFERENTIABLE FRACTIONAL PROGRAMMING:

Mond considered the following primal problem

$$\text{(NFP):} \qquad \text{Maximize } Q(x) = \frac{F(x)}{G(x)} = \frac{f(x) - (x^T Bx)^{1/2}}{g(x) + (x^T Dx)^{1/2}}$$

Subject to,

$h(x) \leq 0$ ,

where f, g and h are differentiable functions from $R^n$ into R, $R^m$ separately, B and D are n x n symmetric positive semi-definite frameworks and G(x) > 0 for all possible x. Under suitable presumptions on f, g and h, Mond [9] set up fundamental and adequate conditions for the presence of an ideal answer for (NFP) and, detailed a double problem to (NFP) and built up suitable duality theorems under convexity presumptions on f, g, and h.

Later Zhang and Mend determined various types of vital and adequate conditions for the presence of an ideal arrangement of (NFP). As an application of these optimality conditions, they figured the first and second request duals given underneath separately and set up suitable duality theorems.

$$(\text{DFD}_1): \quad \text{Minimize } \frac{f(u) - u^T Bv}{g(u) + u^T Dw}$$

## 8. VARIATIONAL PROBLEMS

A variation problem can be considered as a specific instance of an optimal control problem in which the control function is the subordinate of a state function.

Numerically, a variational problem is of the frame:

$$(\text{VP}): \quad \text{Minimize } \int_a^b f(t, x, \dot{x})dt$$

Subject to

$$x(a) = \alpha, \; x(b) = \beta$$

$$g(t, x, \dot{x}) \leq 0, \; t \in I.$$

$$x \in C(I, R^n).$$

where I = [a,b] is a real time interval, x signifies subsidiary of x regarding t, f :I x $R^u$ x $R^n \rightarrow R^m$ and g : I x $R^u$ x $R^n \rightarrow R^m$ are persistently differentiable functions as for every of their contentions; C (I, $R^n$) is the space of persistently differentiable functions x : I→ $R^n$, and is outfitted with the norm $\|x\| = |x\|_\infty$ +

$\|D_x\|_\infty$ where the separation operator D is given by y=Dx<=> x(t)=x(a)+ $\int_a^1 y(s)ds$ aside from at a discontinuities. The accompanying vital conditions for the presence o f an external for (VP) are determined by Valentine,

## 9. CONCLUSION

Be that as it may, in the world of today, the expanded rivalry and buyer demands regularly require optimum arrangements instead of simply doable arrangements. It has been encountered that optimization of design process spares cash for a organization by essentially decreasing the improvement time. In this manner the theory of optimization manages picking the best option among a few choices in the feeling of given capacity with minimum conceivable assets. This creates a class of problems named as mathematical programming problems. The optimum looking for techniques are known as mathematical programming techniques and by and large concentrated as a piece of operations research.

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