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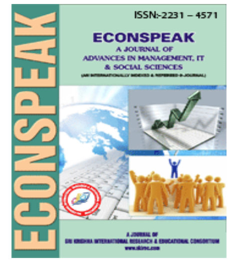

ROLE OF MATHEMATICS IN ECONOMICS

DR. NIDHI SHARMA*

*Assistant Professor,
S.U.S Govt. College, Matak Majri,
Indri, Karnal, India.

ABSTRACT

Role of Mathematics in Economics is the application of mathematical methods to represent economic theories and analyze problems posed in economics. It allows formulation and derivation of key relationships in a theory with clarity, generality, rigor, and simplicity. By convention, the methods refer to those beyond simple geometry, such as differential and integral calculus, difference and differential equations, matrix algebra, and mathematical programming and other computational methods. The mathematization of economics began in earnest in the 19th century. Most of the economic analysis of the time was what would later be called classical economics.In the late 1930s, economists saw the wider use of a broad array of mathematical tools, including convex sets and graph theory. Mathematicians began to discuss economic problems as a means to advance the state of pure mathematics in the same sense that solutions to problems in physics led to advancement in the underlying mathematics.In 1936, the Russian–born economist Wassily Leontief built his model of input-output analysis from the 'material balance' tables constructed by Soviet economists, which themselves followed earlier work by Austrian economists and the physiocratsLinear programming was developed to aid the allocation of resources in firms and in industries during the 1930s in Russia and during the 1940s in the United States. During the Berlin airlift (1948), linear programming was used to plan the shipment of supplies to prevent Berlin from starving after the Communist blockade.[51]Working with Oskar Morgenstern on the theory of games, von Neumann declared that economic theory needed to use functional analytic methods, especially convex sets and topological fixed point theorem, rather than the traditional differential calculus, because the maximum–operator did not preserve differentiable functions.Moreover, differential calculus has returned to the highest levels of mathematical economics, general equilibrium theory(GET), as practiced by the "GET-set" (the humorous designation due to Jacques H. Drèze).

INTRODUCTION

Role of Mathematics in economics is the application of mathematical methods to represent economic theories and analyze problems posed in economics. It allows formulation and derivation of key relationships in a theory with clarity, generality, rigor, and simplicity. By convention, the methods refer to those beyond simple geometry, such as differential and integral calculus, difference and differential equations, matrix algebra, and mathematical programming and other computational methods.

Mathematics allows economists to form meaningful, testable propositions about many wide-ranging and complex subjects which could not be adequately expressed informally. Further, the language of mathematics allows economists to make clear, specific, positive claims about controversial or contentious subjects that would be impossible without mathematics. Much of economic theory is currently presented in terms of mathematical economic models, a set of stylized and simplified mathematical relationships that clarify assumptions and implications.

Broad applications include:

• optimization problems as to goal equilibrium, whether of a household, business firm, or policy maker

• static (or equilibrium) analysis in which the economic unit (such as a household) or economic system (such as a market or the economy) is modeled as not changing

• comparative statics as to a change from one equilibrium to another induced by a change in one or more factors

• dynamic analysis, tracing changes in an economic system over time, for example fromeconomic growth.

Formal economic modeling began in the 19th century with the use of differential calculus to represent and explain economic behavior, such as utility maximization, an early economic application of mathematical optimization. Economics became more mathematical as a discipline throughout the first half of the 20th century, but introduction of new and generalized techniques in the period around the Second World War, as in game theory, would greatly broaden the use of mathematical formulations in economics.

The use of mathematics in the service of social and economic analysis dates back to the 17th century. Then, mainly in German universities, a style of instruction emerged which dealt specifically with detailed presentation of data as it related to public administration. Gottfried Achenwall lectured in this fashion, coining the term statistics.

At the same time, a small group of professors in England established a method of "reasoning by figures upon things relating to government" and referred to this practice as Political Arithmetick. Sir William Petty wrote at length on issues that would later concern economists, such as taxation, Velocity of money and national income, but while his analysis was numerical, he rejected abstract mathematical methodology. Petty's use of detailed numerical data (along with John Graunt) would influence statisticians and economists for some time, even though Petty's works were largely ignored by English scholars.

Modern Mathematical Economics

In the late 1930s, economists saw the wider use of a broad array of mathematical tools, including convex sets and graph theory. Mathematicians began to discuss economic problems as a means to advance the state of pure mathematics in the same sense that solutions to problems in physics led to advancement in the underlying mathematics.

DIFFERENTIAL CALCULUS

Vilfredo Pareto analyzed microeconomics by treating decisions by economic actors as attempts to change a given allotment of goods to another, more preferred allotment. Sets of allocations could then be treated as Pareto efficient (Pareto optimal is an equivalent term) when no exchanges could occur between actors that could make at least one individual better off without making any other individual worse off. Pareto's proof is commonly conflated with Walrassian equilibrium or informally ascribed to Adam Smith's Invisible handhypothesis. Rather, Pareto's statement was the first formal assertion of what would be known as the first fundamental theorem of welfare economics. These models lacked the inequalities of the next generation of mathematical economics.

In the landmark treatise Foundations of Economic Analysis (1947), Paul Samuelson identified a common paradigm and mathematical structure across multiple fields in the subject, building on previous work by Alfred Marshall. Foundations took mathematical concepts from physics and applied them to economic problems. This broad view (for example, comparing Le Chatelier's principle to tâtonnement) drives the fundamental premise of mathematical economics: systems of economic actors may be modeled and their behavior described much like any other system. This extension followed on the work of the marginalists in the previous century and extended it significantly. Samuelson approached the problems of applying individual utility maximization over aggregate groups with comparative statics, which compares two different equilibrium states after an exogenous change in a variable. This and other methods in the book provided the foundation for mathematical economics in the 20th century.

LINEAR MODELS

Restricted models of general equilibrium were formulated by John von Neumann in 1938: Unlike earlier versions, the models of von Neumann had inequality constraints. For his model of an expanding economy, von Neumann proved the existence and uniqueness of an equilibrium using his generalization of Brouwer's fixed point theorem. Von Neumann's model of an expanding economy considered thematrix pencil $A - \lambda B$ with nonnegative matrices A and B; von Neumann sought probability vectors p and q and a positive number λ that would solve the complementarity equation

$$p^T (A - \lambda B) q = 0,$$

along with two inequality systems expressing economic efficiency. In this model, the (transposed) probability vector p represents the prices of the goods while the probability vector q represents the "intensity" at which the production process would run. The unique solution λ represents the rate of growth of the economy, which equals the interest rate. Proving the existence of a positive growth rate and proving that the growth rate equals the interest rate were remarkable achievements, even for von Neumann. Von Neumann's results have been viewed as a special case of linear programming, where von Neumann's model uses only nonnegative matrices. The study of von Neumann's model of an expanding economy continues to interest mathematical economists with interests in computational economics.

INPUT OUTPUT ECONOMICS

In 1936, the Russian–born economist Wassily Leontief built his model of input-output analysis from the 'material balance' tables constructed by Soviet economists, which themselves followed earlier work by Austrian economists and the physiocrats. With his model, which described a system of production and demand processes, Leontief described how changes in demand in one economic sectorwould influence production in another. In practice, Leontief estimated the coefficients of his simple models, to address economically interesting questions. In production economics, "Leontief technologies" produce outputs using constant proportions of inputs, regardless of the price of inputs, reducing the value of Leontief models for understanding economies but allowing their parameters to be estimated relatively easily. In contrast, the von Neumann model of an expanding economy allows for choice of techniques, but the coefficients must be estimated for each technology.

MATHEMATICAL OPTIMIZATION

Linear and nonlinear programming profoundly enriched microeconomics, which had earlier considered only equality constraints. Many of the mathematical economists who received Nobel Prizes in Economics had conducted notable research using linear programming: Leonid Kantorovich, Leonid Hurwicz, Tjalling Koopmans, Kenneth J. Arrow, Paul Samuelson, Robert Solow, and Robert Dorfman. Both Kantorovich and Koopmans acknowledged that George B. Dantzig deserved to share their Nobel Prize for linear programming. Economists who conducted research in nonlinear programming also have won the Nobel prize, notably Ragnar Frisch in addition to Kantorovich, Hurwicz, Koopmans, Arrow, and Samuelson.

LINEAR OPTIMIZATION

Linear programming was developed to aid the allocation of resources in firms and in industries during the 1930s in Russia and during the 1940s in the United States. During the Berlin airlift (1948), linear programming was used to plan the shipment of supplies to prevent Berlin from starving after the Communist blockade.[51]

NONLINEAR PROGRAMMING

Extensions to nonlinear optimization with inequality constraints were achieved in 1951 by Albert W. Tucker and Harold Kuhn, who considered the nonlinear optimization problem:

$$\text{Minimize} \quad f(x)$$

$$\text{subject to:}$$

$$g_i(x) \leq 0, h_j(x) = 0$$

where $f(\cdot)$ is the function to be minimized, where $g_i(\cdot)\ (i = 1, \ldots, m)$ are the functions of the inequality constraints and $h_j(\cdot)\ (j = 1, \ldots, l)$ are the functions of the equality constraints, and where m and l are the number of inequality and equality constraints, respectively. In allowing inequality constraints, the Kuhn–Tucker approach generalized the classic method of Lagrange multipliers, which (until then) had allowed only equality constraints. The Kuhn–Tucker approach inspired further research on Lagrangian duality, including the treatment of inequality constraints. The duality theory of nonlinear programming is particularly satisfactory when applied to convex minimization problems, which enjoy the convex-analytic duality theory of Fenchel and Rockafellar; this convex duality is particularly strong for polyhedral convex functions, such as those arising in linear programming. Lagrangian duality and convex analysis are used daily in operations research, in the scheduling of power plants, the planning of production schedules for factories, and the routing of airlines (routes, flights, planes, crews).

GAME THEORY

Working with Oskar Morgenstern on the theory of games, von Neumann declared that economic theory needed to use functional analytic methods, especially convex sets and topological fixed point theorem, rather than the traditional differential calculus, because the maximum–operator did not preserve differentiable functions. Continuing von Neumann's work in cooperative game theory, game theorists Lloyd S. Shapley, Martin Shubik, Hervé Moulin, Nimrod Megiddo, Bezalel Peleg influenced economic research in politics and economics. For example, research on the fair prices in cooperative games and fair values for voting games led to changed rules for voting in legislatures and for accounting for the costs in public–works projects: For example, cooperative game theory was used in designing the water distribution system of Southern Sweden and for setting rates for dedicated telephone lines in the USA.

Following von Neumann's program, John Nash used fixed–point theory to prove that his noncooperative games and his bargaining problems have equilibria. For decades, noncooperative game theory was adopted by greater numbers of microeconomists, whose work illuminated problems of industrial organization. In 1994, Nash, John Harsanyi, and Reinhard Selten received the Nobel Memorial Prize in Economic Sciences their work on non–cooperative games; Harsanyi and Selten were rewarded for their work on repeated games..

FUNCTIONAL ANALYSIS

Following von Neumann's program, Kenneth Arrow and Gérard Debreu formulated abstract models of economic equilibria using convex sets and fixed–point theory. Introduced the Arrow-Debreu model in 1954, they proved the existence (but not the uniqueness) of an equilibrium and also proved that every Walras equilibrium is Pareto efficient; in general, equilibria need not be unique. In their models, the ("primal") vector space represented quantitites while the "dual" vector space represented prices.

In Russia, the mathematician Leonid Kantorovich developed economic models in partially ordered vector spaces, that emphasized the duality betweeen quantities and prices. Oppressed by communism, Kantorovich renamed prices as "objectively determined valuations" which were abbreviated in Russian as "o. o. o.", alluding to the difficulty of discussing prices in the Soviet Union.

Even in finite dimensions, the concepts of functional analysis have illuminated economic theory, particularly in clarifying the role of prices as normal vectors to a hyperplane supporting a convex set, representing production or consumption possibilities. However, problems of describing optimization over time or under uncertainty require the use of infinite–dimensional function spaces, because agents are chosing among functions or stochastic processes.

VARIATIONAL CALCULUS AND OPTIMAL CONTROL

The problem of finding optimal functions is studied in variational calculus and in optimal control theory. Before the Second World War,Frank Ramsey and Harold Hotelling used the calculus of variations to find optimal solutions to dynamic economics problems.

Optimal control theory began to be used in addressing dynamic problems in economics, especially the economic growth models, soon after Richard Bellman's work on dynamic programming and after the publication of the English translation of the book by Pontryagin et al. Applications of optimal control theory include those in economic growth, finance, inventories, and production for example., for example.

DIFFERENTIAL RENAISSANCE

As discussed below, following John von Neumann's break-throughs in economics, and particularly after his introduction of functional analysis and topology in economic theory, advanced mathematical economics reduced its emphasis on differential calculus. In general equilibrium theory, mathematical economists used general topology, convex geometry, and optimization theory more than differential calculus, because the approach of differential calculus had failed to establish the existence of an equilibrium. However, the decline of differential calculus should not be exaggerated, because differential calculus is still used in graduate training and applications. Moreover, differential calculus has returned to the highest levels of mathematical economics, general equilibrium theory (GET), as practiced by the "GET-set" (the humorous designation due to Jacques H. Drèze).

In the 1960s and 1970s, however, Gérard Debreu and Stephen Smale led a revival of the use of differential calculus in mathematical economics. In particular, they were able to prove the existence of a general equilibrium, where earlier writers had failed, because of their novel mathematics: Baire category from general topology and Sard's lemma from differential topology. Other economists asssociated with the use of differential analysis include Egbert Dierker, Andreu Mas-Colell, and Yves Balasko. These advances have changed the traditional narrative of the history of mathematical economics, following von Neumann, which celebrated the abandonment of differential calculus.

ECONOMETRICS

Between the world wars, advances in mathematical statistics and a cadre of mathematically trained economists led to econometrics, which was the name proposed for the discipline of advancing economics by using mathematics and statistics. Within economics, "econometrics" has often been used for statistical methods in economics, rather than mathematical economics. Statistical econometrics features the application of linear regression and time series analysis to economic data.

Ragnar Frisch coined the word "econometrics" and helped to found both the Econometric Society in 1930 and the journal Econometricain 1933. A student of Frisch's, Trygve Haavelmo published The Probability Approach in Econometrics in 1944, where he asserted that precise statistical analysis could be used as a tool to validate mathematical theories about economic actors with data from complex sources. This linking of statistical analysis of systems to economic theory was also promulgated by the Cowles Commission (now the Cowles Foundation) throughout the 1930s and 1940s.

he published a dynamic "moving equilibrium" model designed to explain business cycles—this periodic variation from overcorrection in supply and demand curves is now known as the cobweb model. A more formal derivation of this model was made later by Nicholas Kaldor, who is largely credited for its exposition.[]

CRITICISMS AND DEFENCES

ADEQUACY OF MATHEMATICS FOR QUALITATIVE AND COMPLICATED ECONOMICS

Friedrich Hayek contended that the use of formal techniques projects a scientific exactness that does not appropriately account for informational limitations faced by real economic agents.

Heilbroner stated that "some/much of economics is not naturally quantitative and therefore does not lend itself to mathematical exposition.

TESTING PREDICTIONS OF MATHEMATICAL ECONOMICS

Philosopher Karl Popper discussed the scientific standing of economics in the 1940s and 1950s. He argued that mathematical economics suffered from being tautological. In other words, insofar that economics became a mathematical theory, mathematical economics ceased to rely on empirical refutation but rather relied on mathematical proofs and disproof. According to Popper, falsifiable assumptions can be tested by experiment and observation while unfalsifiable assumptions can be explored mathematically for their consequences and for their consistency with other assumptions.

Sharing Popper's concerns about assumptions in economics generally, and not just mathematical economics, Milton Friedman declared that "all assumptions are unrealistic". Friedman proposed judging economic models by their predictive performance rather than by the match between their assumptions and reality.

MATHEMATICAL ECONOMICS AS A FORM OF PURE MATHEMATICS

It is a great fault of symbolic pseudo-mathematical methods of formalising a system of economic analysis … that they expressly assume strict independence between the factors involved and lose their cogency and authority if this hypothesis is disallowed; whereas, in ordinary discourse, where we are not blindly manipulating and know all the time what we are doing and what the words mean, we can keep 'at the back of our heads' the necessary reserves and qualifications and the adjustments which we shall have to make later on, in a way in which we cannot keep complicated partial differentials 'at the back' of several pages of algebra which assume they all vanish. Too large a proportion of recent 'mathematical' economics are merely concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols.

DEFENSE OF MATHEMATICAL ECONOMICS

In response to these criticisms, Paul Samuelson argued that mathematics is a language, repeating a thesis of Josiah Willard Gibbs. In economics, the language of mathematics is sometimes necessary for representing substantive problems. Moreover, mathematical economics has led to conceptual advances in economics. In particular, Samuelson gave the example of microeconomics, writing that "few people are ingenious enough to grasp [its] more complex parts... without resorting to the language of mathematics, while most ordinary individuals can do so fairly easily with the aid of mathematics."

Some economists state that mathematical economics deserves support just like other forms of mathematics, particularly its neighbors in mathematical optimization and mathematical statistics and increasingly in theoretical computer science. Mathematical economics and other mathematical sciences have a history in which theoretical advances have regularly contributed to the reform of the more applied branches of economics. In particular, following the program of John von Neumann, game theory now provides the foundations for describing much of applied economics, from statistical decision theory (as "games against nature") and econometrics to general equilibrium theory and industrial organization. In the last decade, with the rise of the internet, mathematical economicists and optimization experts and computer scientists have worked on problems of pricing for on-line services --- their contributions using mathematics from cooperative game theory, nondifferentiable optimization, and combinatorial games.

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