



**THE RELATIONSHIP BETWEEN RANK SIZE RULE AND EXPONENTIAL RURAL TALUK SIZE  
DISTRIBUTION OF SC AND ST POPULATION**

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**Abstract**

Rural SC and ST population data in Tamil Nadu State provided by 2001 and 2011 census of India (Directorate of census operations, Tamil Nadu, 2011) are proposed in the present investigation of Rural Taluk Size Distribution. The rural Scheduled Castes (SC) population of Tamil Nadu in 2001 census is 8.309 million and in 2011 as 9.5 million. The rural Scheduled Tribes (ST) population of Tamil Nadu in 2001 census is 0.551 million and in 2011 as 0.66 million. It shows the increasing nature of rural SC and ST population and it motivates to study the rural taluk population using the exponential model. Exponential model for rural taluk size distribution is proposed to establish the theoretical relationship between the exponential model for Rural Taluk Size Distribution and strong expected rank size rule. Empirical evidences are necessary in support of the theoretical relationship between the Exponential Rural Taluk Size Distribution and strong expected rank size rule.

**Introduction**

Rural place located outside the city or town with population less than or equal to five thousand is called as a rural area. Classification of rural taluk size with respect to the size is called as Rural Taluk Size Distribution. It has the substantive interest in many socio and demographic fields. The SC and ST population in Tamil Nadu has been rapidly increasing due to natural growth. Established studies of population are reviewed critically and observed that A. Okabe (1979), B. Renganathan (1986, 2004) studied the urban population, the relationship between the rank size rule and the city size distribution analytically and empirically. Further, B. Renganathan (2005) studied the concentration of rural population in Tamil Nadu state using probabilistic model. The review of literature papers showed that none of the studies deals with Rural Taluk Size Distribution of SC and ST population in Tamil Nadu State. The rural SC and ST population census data of 2011 were having an increasing tendency at Tamil Nadu level. The empirical distribution of rural taluk size distribution in 2001 and 2011 census data showed the skew in nature. Hence it motivated to propose the Exponential model for rural taluk size is proposed to establish its theoretical relationship with strong expected rank size rule. Empirical evidences are necessary in support of the theoretical relationship



between the Exponential model for rural taluk size and strong expected rank size rule. Hence the present investigation is undertaken.

**Objective**

To evaluate the relationship between the Exponential model and the strong expected rank size rule with the empirical evidence.

**Data Source**

Data on census 2001 and 2011 SC and ST population (Directorate of census operations, Tamil Nadu, 2011) are applied for analyzing the nature of taluk size distribution in Tamil Nadu state.

**Order statistics**

The function  $X_{(k)}$  of  $(X_1, X_2, \dots, X_n)$  that takes on the value  $x_{(k)}$  in each possible sequence  $(x_1, x_2, \dots, x_n)$  of values assumed by  $(X_1, X_2, \dots, X_n)$  is known as the  $k^{\text{th}}$  order statistic (or) statistic of order  $k$ .  $\{X_{(1)}, X_{(2)}, \dots, X_{(n)}\}$  is called the set of order statistics for  $(X_1, X_2, \dots, X_n)$ .

**Exponential model**

Let  $X$  be a random variable representing rural taluk size and it has the probability density function as,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}; & x > 0, \\ 0 & ; \text{ otherwise} \end{cases}$$

where  $\lambda > 0$  is the parameter of the distribution and is estimated by the method of moments.

The distribution function of the exponential model is,

$$F_X(x) = P(X \leq x) = 1 - e^{-\lambda x} \quad \text{for all } x > 0; \lambda > 0$$

The estimate of the parameter  $\lambda$  of the exponential model is obtained as,  $\hat{\lambda} = \frac{1}{\bar{X}}$

**Beta distribution**

Let  $X$  be a random variable representing rural taluk size and it has the Beta one distribution with parameters  $(r, n-r+1)$ , then its probability density function is stated as,

$$f(x) = \frac{1}{B(r, n-r+1)} (x)^{r-1} (1-x)^{n-r}; \quad B(r, n-r+1) > 0, \quad 0 < x < 1$$

where  $B(r, n-r+1)$  is the Betafunction.

The mean and variance of the rural taluk size distribution are obtained as,

$$E(X) = \frac{r}{n+1}, \quad V(X) = \frac{r(n-r+1)}{(n+1)^2(n+2)}$$

**Distribution of  $r^{\text{th}}$  order statistics**

Let  $X_{(r)}$  be the  $r^{\text{th}}$  order statistics, then its probability density function is stated as,

$$g_r(x_r) = \frac{n!}{(r-1)!(n-r)!} [F(x_r)]^{r-1} [1-F(x_r)]^{n-r} f(x_r)$$



where  $F$  is the common distribution function of  $X$ .

**Rank Size Rule**

The relation,

$$X_{(r)} R_{(r)}^q = C, \text{ for all } r = 1, 2, 3 \dots n,$$

where  $n$  is the no. of taluks,

$X_{(r)}$  is the size of the  $r^{\text{th}}$  ranked taluks,  $R_{(r)}$  is the rank of the  $r^{\text{th}}$  taluks,  $C$  and  $q$  are constants.

is called as rank size rule.

Rank size rule has been described probabilistically through an application of order statistics to study the relationship between rank size rule and Rural Taluk Size Distribution.

At a certain given point of time, 'n' taluks in a state are ordered or ranked according to their sizes  $X = [X_{(1)}, X_{(2)}, \dots, X_{(n)}]$ . A. Okabe (1979) assumed that the set of observed values of  $X$  consists of 'n' taluk size values. These values are sampled according to the same RTSDF ( $X$ ). As the observed Taluk Size values are sampled, ranked 'n' Taluk Size values  $X = [X_{(1)}, X_{(2)}, \dots, X_{(r)}, \dots, X_{(n)}; X_{(1)} < X_{(2)} < \dots < X_{(n)}]$  are probabilistic. Then expected Taluk Size is obtained as,

$$E(X) = \{E[X_{(1)}] E[X_{(2)}] \dots E[X_{(r)}] \dots [E[X_{(n)}]]\}$$

By using the expected Taluk Size, the Rank Size Rule is recalled as expected Rank Size Rule.

i.e.,  $E[X_{(r)}] R_{(r)}^q = C, \text{ for all } r = 1, 2, \dots, n,$

**Strong Expected Rank Size rule**

A. Okabe (1979) established Strong Expected Rank Size Rule as,

$$E[X_{r/n}] R(r) = C(n) \text{ when } r = r^*, r^*+1, \dots, n$$

$$n = r^*+1, r^*+2, \dots$$

where  $r^*$  is the minimum positive integer such that  $R(r) > 0$ ,  $R(r)$  rank function, is dependent of  $n$ , and  $C(n)$  is a constant function of 'n'.

The relationship between Rural Taluk Size Distribution and the Strong expected rank size rule is explained in the following section.

**Exponential Rural Taluk Size Distribution and Strong expected rank size rule**

Let  $X_1, X_2, X_3, \dots, X_n$  be the rural taluk size random variable sampled from the distribution function  $F$  and rural taluk size are independent identically distributed with exponential probability density function,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}; & x > 0, \lambda > 0 \\ 0 & ; \text{ otherwise} \end{cases}$$



when  $X_{(r)}$  be the  $r^{th}$  order Statistics, its probability density function corresponding to  $X_{(r)}$  is obtained as,

$$f [X_{(r)}] = \begin{cases} \lambda e^{-\lambda X_{(r)}}; & X_{(r)} > 0, \lambda > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The distribution function of  $X_{(r)}$  is stated as,

$$F[X_{(r)}] = \int_0^{X_{(r)}} f[X_{(r)}] dx_{(r)} \\ = \int_0^{X_{(r)}} \lambda e^{-\lambda X_{(r)}} dx_{(r)}$$

The  $r^{th}$  order Rural Taluk Size Distribution is stated as,

$$f [X_{(r)}] = \frac{1}{B(r, n-r+1)} f [x_{(r)}] [F(x_{(r)})]^{r-1} [1-F(x_{(r)})]^{n-r}$$

Let  $F [X_{(r)}] = U_{(r)}$

The  $r^{th}$  order Rural Taluk Size Distribution is,

$$g [x_{(r)}] = \frac{1}{B(r, n-r+1)} f [x_{(r)}] [U_{(r)}]^{r-1} [1-U_{(r)}]^{n-r} \frac{1}{f [X_{(r)}]} \\ = \frac{1}{B(r, n-r+1)} [U_{(r)}]^{r-1} [1-U_{(r)}]^{n-r}, \quad 0 < U_{(r)} < 1$$

This is called as a beta one distribution with parameters  $(r, n-r+1)$

$$E [U_{(r)}] = \frac{r}{r+n-r+1} = \frac{r}{n+1}$$

$$V [U_{(r)}] = \frac{r(n-r+1)}{(n+1)^2(r+n-r+1+1)} \\ = \frac{r(n-r+1)}{(n+1)^2(n+2)}$$

$$E [U_{(r)}] = \frac{r}{n+1} = E [F [X_{(r)}]] \quad [\because U_{(r)} = F [X_{(r)}]]$$

By using the probability integral transformation,

$$F^{-1}[U_{(r)}] = x_{(r)} \quad [\because U_{(r)} = F [x_{(r)}]]$$

$$x_{(r)} = F^{-1} [U_{(r)}]$$

$$E [X_{(r)}] = E [F^{-1}(U_{(r)})] \\ \leq F^{-1}[E (U_{(r)})] \quad [\because \text{Jensen's inequality } f [E(X)] \leq E [f(x)]]$$

where,  $f$  is a convex and monotonic increasing function]

$$= F^{-1} \left( \frac{r}{n+1} \right) = \frac{r}{n+1} \\ = \frac{g(r)}{n+1}, \quad \text{where, } g(r) = r$$

$$\therefore E [X_{(r)}] \leq \frac{g(r)}{n+1}$$

when  $F^{-1}[U_{(r)}]$  is linear function in  $U_{(r)}$ ,  $F^{-1}(\frac{1}{n+1})$  is linear,  $g(r) = r$

$$E [X_{(r)}] = \frac{r}{n+1}$$

$$E [X_{(r)}] \cdot \frac{1}{r} = \frac{1}{n+1}$$

$$E [X_{(r)}] R(r) = c (n)$$

$$\text{where } R(r) = \frac{1}{r}, \quad c (n) = \frac{1}{n+1}$$

The strong expected rank size rule,

$$E [X_{(r)}] R(r) = c (n)$$

$$\text{where } R(r) = \frac{1}{r}, \quad c (n) = \frac{1}{n+1}$$

is satisfied by the Exponential Rural Taluk Size Distribution because

$$F^{-1}[U_{(r)}]: \inf \{ X_{(r)}: F[X_{(r)}] \geq \frac{r}{n+1} \}$$

is satisfied by Exponential Rural Taluk Size Variable.

Thus, the strong expected rank size rule has been satisfied by exponential Rural Taluk Size Distributions.

### Empirical results

Rural Taluk Size Distribution of SC and ST population based on 2001 and 2011 census resembles the skew distribution as seen in Table - 1.

**Table -1                      Empirical distribution of rural taluk size-2001 and 2011  
   SC and ST population**

Rural Taluk Size (in'000)	Number of rural taluks	
	2001	2011
0-20	33	34
20-40	7	70
40-60	44	47
60-80	21	22
80-100	17	16
100-120	5	11
120-140	2	4
>140	1	2
<b>Total</b>	<b>200</b>	<b>206</b>

### Exponential Model

Exponential Model is fitted using **2001** population data given in the Table -1 as follows:

The mean of the empirical rural taluk size distribution is obtained as,

$$\bar{X} = 44$$

The estimates of the parameter of the exponential model is obtained as,

$$\hat{\lambda} = 0.0227 > 0$$

The fitted model is stated as,

$$f(x) = \begin{cases} (0.0227)e^{(-0.0227)x} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The fitted function  $f(x)$  of rural taluks is obtained and presented in Table -2 below:

**Table -2      Observed and the expected distribution of rural taluks for  
SC and ST Population – 2001**

Rural Taluk Size (in'000)	Observed number of rural taluks	Probability values  f(x)
0-20	33	0.3649
20-40	7	0.2317
40-60	44	0.1472
60-80	21	0.0935
80-100	17	0.0594
100-120	5	0.0377
120-140	2	0.0239
>140	1	0.0417
<b>Total</b>	<b>200</b>	<b>1.0000</b>

\* Exponential Model is fitted using **2011** population data given in the Table -1 as follows:

The mean of the empirical rural taluk size distribution is obtained as,

$$\bar{X} = 47.5728,$$

The estimates of the parameter of the exponential model is obtained as,

$$\hat{\lambda} = 0.0210 > 0$$

The fitted model is stated as,

$$f(x) = \begin{cases} (0.021)e^{(-0.021)x} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

The fitted function f(x) of rural taluks is obtained and presented in Table -3 below:

**Table -3                      Observed and the expected distribution of rural taluks for**

**SC and ST Population – 2011**

<b>Rural Taluk Size (in'000)</b>	<b>Observed number of rural taluks</b>	<b>Probability values f(x)</b>
0-20	34	0.3429
20-40	70	0.2253
40-60	47	0.1481
60-80	22	0.0973
80-100	16	0.0639
100-120	11	0.0420
120-140	4	0.0276
>140	2	0.0529
<b>Total</b>	<b>206</b>	<b>1.0000</b>

**Exponential Rural Taluk Size Distribution - SC & ST Population 2001**

The  $r^{\text{th}}$  order exponential distribution function  $F [x_{(r)}]$  values have been computed using the information given in Table -2 and presented in Table -4 below.



**Table -4      Computation of Exponential  $F[x_{(r)}]$  values- SC and ST**

**Population 2001**

Rural Taluk Size (in'000)	Rank of the rural taluk (r)	F[x]	1-F[x]=R(x)	$X_{(r)}$	$F[x_{(r)}]$
0-20	1	0.3649	0.6351	0.0567	0.3647
20-40	2	0.5966	0.4034	0.1864	0.7288
40-60	3	0.7438	0.2562	0.4134	0.9163
60-80	4	0.8373	0.1627	0.7766	0.9794
80-100	5	0.8967	0.1033	1.3441	0.9954
100-120	6	0.9344	0.0656	2.2521	0.9988
120-140	7	0.9583	0.0417	3.8407	0.9995
>140	8	1.0000	0.0000	$\infty$	1.0000

The probability integral transformation is verified empirically as follows.

$$F^{-1} [F (x_{(r)})]: F^{-1} [U_{(r)}] = \inf \{X_{(r)}: F [x_{(r)}] \geq \frac{r}{n+1} \}$$

When r = 1,     $\{X_{(1)}: F [x_{(1)}] = 0.3647 \geq \frac{1}{9} = 0.1111\}$

When r = 2,     $\{X_{(2)}: F [x_{(2)}] = 0.7288 \geq \frac{2}{9} = 0.2222\}$

When r = 3,     $\{X_{(3)}: F [x_{(3)}] = 0.9163 \geq \frac{3}{9} = 0.3333\}$

When r = 4,     $\{X_{(4)}: F [x_{(4)}] = 0.9794 \geq \frac{4}{9} = 0.4444\}$

When r = 5,     $\{X_{(5)}: F [x_{(5)}] = 0.9954 \geq \frac{5}{9} = 0.5555\}$

When r = 6,     $\{X_{(6)}: F [x_{(6)}] = 0.9988 \geq \frac{6}{9} = 0.6666\}$

When r = 7,     $\{X_{(7)}: F [x_{(7)}] = 0.9995 \geq \frac{7}{9} = 0.7777\}$



When  $r = 8$ ,  $\{X_{(8)}: F [X_{(8)}] = 1.0000 \geq \frac{8}{9} = 0.8888\}$

It implies that Strong expected rank size rule is satisfied by the exponential distribution of rural taluk size.

**Exponential Rural Taluk Size Distribution – SC & ST Population 2011**

The  $r^{\text{th}}$  order exponential distribution function  $F [x_{(r)}]$  values have been computed using the information given in Table -3 and presented in Table -5 below:

**Table -5                      Computation of Exponential  $F[x_{(r)}]$  values-SC and ST population 2011**

Rural Taluk Size (in'000)	Rank of the rural taluk (r)	F[x]	1-F[x]=R(x)	$X_{(r)}$	F[x <sub>(r)</sub> ]
0-20	1	0.3429	0.6571	0.0525	0.3429
20-40	2	0.5682	0.4318	0.1725	0.7010
40-60	3	0.7163	0.2837	0.3825	0.8992
60-80	4	0.8136	0.1864	0.7185	0.9725
80-100	5	0.8775	0.1225	1.2434	0.9931
100-120	6	0.9195	0.0805	2.0832	0.9981
120-140	7	0.9471	0.0529	3.5529	0.9992
>140	8	1.0000	0.0000	$\infty$	1.0000

The probability integral transformation is verified empirically as follows.

$$F^{-1} [F (x_{(r)})]: F^{-1} [U_{(r)}] = \inf \{X_{(r)}: F [X_{(r)}] \geq \frac{r}{n+1}\}$$

When  $r = 1$ ,  $\{X_{(1)}: F [X_{(1)}] = 0.3429 \geq \frac{1}{9} = 0.1111\}$

When  $r = 2$ ,  $\{X_{(2)}: F [X_{(2)}] = 0.7010 \geq \frac{2}{9} = 0.2222\}$

When  $r = 3$ ,  $\{X_{(3)}: F [X_{(3)}] = 0.8992 \geq \frac{3}{9} = 0.3333\}$

When  $r = 4$ ,  $\{X_{(4)}: F [X_{(4)}] = 0.9725 \geq \frac{4}{9} = 0.4444\}$

When  $r = 5$ ,  $\{X_{(5)}: F [X_{(5)}] = 0.9931 \geq \frac{5}{9} = 0.5555\}$

When  $r = 6$ ,  $\{X_{(6)}: F [X_{(6)}] = 0.9981 \geq \frac{6}{9} = 0.6666\}$

When  $r = 7$ ,  $\{X_{(7)}: F [X_{(7)}] = 0.9992 \geq \frac{7}{9} = 0.7777\}$

When  $r = 8$ ,  $\{X_{(8)}: F [X_{(8)}] = 1.0000 \geq \frac{8}{9} = 0.8888\}$



It implies that Strong expected rank size rule is satisfied by the exponential distribution of rural taluk size.

Since the probability integral transformation is verified empirically, exponential Rural Taluk Size Distribution based on SC & ST population data in the year 2001 & 2011 satisfies the strong expected rank size rule.

### Conclusion

The Exponential model is fitted to the empirical rural taluk size distribution based on the 2001 and 2011 census data for SC and ST population. A Rural Taluk Size Distribution is related to Rank Size Rule in terms of Strong expected Rank Size rule. It is shown that Strong expected Rank Size rule is satisfied by Exponential Rural Taluk Size Distribution. Empirically proved that the exponential model for Rural Taluk Size Distribution satisfy the strong expected rank size rule. Exponential model confirm the real distribution of rural taluk size. The present investigation suggests to the future researchers for analyzing the nature of rural taluk size in all states of India.

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